

## **METHODS OF CLASSIFICATION MODELS FOR ENTERPRISES INSOLVENCY PREDICTION**

Adam Waszkowski

Warsaw University of Life Sciences – SGGW

**Abstract.** The thesis attempts to create models of standard classification which would enable to predict a bankruptcy of enterprises. Economic aspects of bankruptcy have been presented as well as causes of it. The attention has been devoted to the methods of bankruptcy prediction with emphasis on the discriminant analysis and logit models. The thesis contains a review of literature concerning existing models of early warning. Description and the estimation's results of own models of bankruptcy have been included in the chapter based on research. It has also presented results of the verification of obtained functional models based on a sample validation.

**Key words:** bankruptcy prediction, classification models, discriminant analysis, logit model

### **INTRODUCTION**

The theory of company life cycle presents the stages in the development of individuals, which include growth, relative stability and the end-stage phase which may lead to a bankruptcy. Duration as well as a course of the various stages are features typical for a single unit in economy, dependent on many factors, formed inside the company (organizational structure, management) and by the macro environment. Although the theory indicates the inevitability of end-stage phase, the bankruptcy may be moved in time, affected its course and prevented accordingly early. Bankruptcy leads to a verification of the effectiveness of the entity in a market economy and a better location of resources “transferring” them from inefficient companies into the thriving ones. On the other hand, the bankruptcy of every individual creates in the economy a rise of additional costs, which are beard by others. The deterioration of the financial situation itself is a slow process and the first of its symptoms can be detected even several years in advance. Warnings about the potential treat of bankruptcy gives the company an opportunity to take action

to protect it from liquidation. Researches in this area are generally kept in two directions. The first concerns the determination of insolvency symptoms. Literature provides information (cf. Mączyńska, Zawadzki [2006]) that the most important of them are liquidity lowering, profitability reduction and a clear drop in sales. These symptoms can be determined by analyzing financial statements. An important role is played by qualitative factors such as: managerial instability or lack of long-term business plans. The second line of research concerns the design, development and validation of quantitative bankruptcy prediction models. The first of such an attempt was launched in the sixties by Altman [1968]. In Poland in 1990 the occurrence of bankruptcy did not happen. Centrally planned economy did not allow a possible review of the effectiveness of enterprises and the life cycle was based on the non-economic condition, often by the political nature. Only later the period of transformation have affected the allocation of resources, which contributed to a wave of bankruptcies.

## OBJECTIVE AND METHOD OF RESEARCH

The aim of the study was to develop a standard classification models that can be used to predict the bankruptcy of enterprises. These models allow the separation from a large population both wealth entities, characterized by the proper financial standing, good organization and efficient management as well as those with no prospect survival. There have been two sets used for models estimation. First are the data from 41 companies-bankrupts collected and compiled on the basis of the failed company's acts of the District Court for the Capital City of Warsaw in Warsaw, which in 2008–2009 declared them bankrupt. The second group includes financial ratios of 41 companies in good financial standing, which carried their businesses throughout the whole 2010. This data has been selected from the Notoria site (version 17.70). Obtained data allowed the estimation of classification models for predicting corporate bankruptcy. To validation of the acquired models there were used an expert selected try (different from the standard try) numbering 8 failed companies and 8 healthy from Notoria site. A unique, rather large for Polish standards set of data have been formed in this way, including 82 enterprises.

The study shows that in the current economic situation of Polish market, a combination of financial ratios analysis, discriminant analysis and functional models of dichotomous variables leads to the construction of tools for bankruptcy prediction.

Discriminant analysis makes possibility to design a mathematical formula which indicates appurtenance to the different classes (the bankrupt company, and a company with good financial standing). The appurtenance rule is the linear combination of financial indicators, which can be written this way:

$$LFD = \lambda_0 + \lambda^T \cdot x, \quad (1)$$

where:  $x$  is a vector of features (financial ratios describing the enterprise),  $\lambda_0$  are vectors of discriminant function ratios. The LFD ratio is estimated so as to maximize the quotient of between-group function appurtenance variance to variance of intra-group. Formula's that determinates the estimation of LFD parameters shows A. Maddala [2004].

Verification of ability of the company's classification into groups by any variable which is defined as discriminatory power is based on a statistical analysis of two statistics:

- Partial Wilks' ratio, defined by a formula:

$$\lambda^{cz} = \frac{\lambda^1}{\lambda^0}, \quad (2)$$

where:  $\lambda^1$  means the value of Wilks' lambda ratio for the model after the introduction of a variable into it,  $\lambda^0$  means the ratio of Wilks' lambda for the model before the introduction of the highlighted variable.

Ratio corresponds to the statistics  $F$  given by the formula:

$$F = \frac{\frac{1 - \lambda^{cz}}{K - 1}}{\frac{\lambda^{cz}}{N - I - K}}, \quad (3)$$

where:  $N$  is the total number of objects in the sample,  $I$  – the number of considered population and  $K$  – the number of variables

- Tolerance factor  $T$ , which is defined as:

$$T = 1 - R_k^2, \quad (4)$$

where:  $R_k^2$  denotes the ratio of multiple correlation between a variable and the other functions occurring in the LFD.

Models which are also used in the bankruptcy forecast are qualitative variables models in which the most popular is the logit model. Their broad overview can be found in the work edited by Gruszczyński [2010]. For the logit model the linear combination of features is transformed by the logistic function. The logit model is as follows:

$$LOG = \frac{1}{1 + e^{-z}} = \frac{e^z}{e^z + 1}, \quad (5)$$

where:  $z = b_0 + b^T x$ .

The logit model (LOG) has the advantage over the linear discriminant function (LFD), that the variability range of estimated depended variable is in a range  $\langle 0, 1 \rangle$  which corresponds to the definition of probability and it the basis for interpretation in the category of bankruptcy and the correct financial condition. For a linear discriminant function a range of variability of the endogenous variable can include a whole set of actual numbers.

## REVIEW OF ENTERPRISE INSOLVENCY CLASSIFICATION MODELS

Discriminant analysis to predict bankruptcy has been used for the first time by Altman [1968]. He built four versions of the Z-score bankruptcy predictor model, which included data from 33 of 'healthy' enterprises and 33 of those where the bankruptcy were declared. The model was characterized by a highly effective prediction (95% of accurate forecast for the year before announcement of bankruptcy, 72% for two years and 48% for the

three-year time horizon). Tests were then followed by numerous authors developing classification models for the economies of different countries and using more sophisticated methods of multivariate data analysis (review of the literature devoted to this subject is presented in the work of Kisielińska [2008] as well as Kisielińska and Waszkowski [2010]). Changes in the Polish economy in the 90s caused the interest of Polish economists of methods to detect a threat of bankruptcy. Many authors have pointed in their works (Stasiewski [1996], Gasza [1997], Rogowski [1999], Koralun-Bereźnicka [2006]) the impossibility of direct adaption of Altmans' models on the Polish market, which resulted of original ways of early warnings elaboration.

Hołda [2011] analyzed 40 failed enterprises and 40 with a good financial situation. Data concerning the objects came from the years 1993–1996. In the prediction models there were following ratios included:

- PWP (basic liquidity ratio) = current assets / current liabilities,
- SZ (debt Ratio) = total liabilities / total assets,
- ZM (profitability of assets) = net financial result / average total assets,
- WOZ (market rate liabilities) = average condition of short-term liabilities / (operating expenses – other operating expenses),
- RM (assets rotation) = total revenue / average total assets.

The linear discriminant function, which were elaborated looks as following:

$$ZH = 0,605 + 0,681 \cdot PWP - 0,0196 \cdot SZ + 0,00969 \cdot ZM + 0,000672 \cdot WOZ + 0,157 \cdot RM$$

The model yielded an aggregate percentage of correct classification on 92,5% (the 95% of the bankruptcies and 90% of companies in good condition were recognized correctly).

Gajdek and Stosa's [2003] early warning system was developed to assess the financial condition of companies listed on the Warsaw Stock Exchange. This model was estimated on the basis of sample consisting of 17 fallen companies and 17 'healthy' companies with a similar business profile. The share of correct diagnoses in the research sample was equal to 100% and the linear discriminant model adapted to the following form:

$$Z = -0,0005 \cdot X1 + 2,0552 \cdot X2 + 1,7260 \cdot X3 + 0,1155 \cdot X4$$

In this model the following ratios has been included:

- X1 = current liabilities / production costs of sold,
- X2 = net outcome / total assets,
- X3 = gross outcome / net revenue from sales,
- X4 = total assets / total liabilities.

Gruszczyński [2003] conducted a study based on 200 financial statements, which were used to select 23 companies in incorrect financial situation and 23 companies of good standing. On their basis the binomial logit models were estimated, the response variable is a dichotomous variable. To the construction of the models there were following ratios used:

- ROA (return on assets) = net profit / assets,
- R1 (gross margin) = gross profit / net revenue from sales,
- A2 (obligations trading) = production costs of sold / current liabilities,
- Z1 (the assets debt ratio) = total liabilities / assets,
- W19 = inventories / net revenue from sales.

The assessments of parameters of chosen models as well as the accuracy of forecast are presented in Table 1.

Table 1. Gruszczyński binomial logit models  
Tabela 1. Dwumianowe modele logitowe Gruszczyńskiego

Model	Parameters	Parameter rate	Forecast accuracy ( $y_i = 0$ ) [ % ]	Forecast accuracy ( $y_i = 1$ ) [ % ]
MLD1	constant	1,3508	86,96	86,96
	ROA1	7,5153		
	Z1	-6,1903		
MLD2	constant	0,3133	82,61	86,96
	ROA1	8,7592		
	W19	-8,0069		
MLD3	constant	4,3515	91,30	95,65
	R1	22,8748		
	Z1	-5,5926		
MLD4	constant	-4,7238	86,96	86,96
	R1	16,1075		
	A2	0,5761		

Source: Own work based on Gruszczyński [2003], p. 17–19.

Źródło: Opracowanie własne na podstawie Gruszczyński [2003], s. 17–19.

## RESEARCH RESULTS

From the standpoint of classification models construction, arguments seen as being financial ratios, which will be included in the early warning systems, must meet certain demands. The most important one concerns an absolute lack of mutual correlation between features. Not meeting this demand causes the wrong conditional of variance-correlation matrix, which in consequence unable an estimation of models parameters. An important role is played also by a discriminatory ability of the individual ratios between the classes. The bigger ability is, the better is the classification of the 'healthy' and bankrupt companies. Moreover the features included in the model must have a sufficiently high volatility. The next step in the selection of variables was the elimination of quasi-permanent features that do not increase the cognitive value of the model. For the ratio used to construct models, the distribution of its value in a population of individuals at risk of bankruptcy differs systematically from the distribution of the population of those with proper standing. Therefore for practical reasons it is assumed that the distribution in both populations is the same, the only difference is in parameters describing it. In empirical studies it is assumed a-priori that the analyzed features are normally distributed. At the significance level of 5% there was a hypothesis rejected for the each of the indicators concerning the normality of empirical distribution function, having in mind simultaneously that in many researches the assumption of normality was not fulfilled, and yet the satisfactory results of classification were obtained (i.e. Hadasik [1998]).

Finally for further analyze there was a set of 15 ratios chosen:

- OPZD (load of enterprise by current liabilities) = current liabilities / total liabilities,
- ROA (return on assets) = net profit / total assets,
- RS (return on sales) = net profit / sales income,
- UNAO (share of liabilities in total assets) = receivables / total assets,
- UNSMAO (share of intangible assets in total assets) = intangible assets / total assets,
- URSKAO (share of tangible assets in total assets) = tangible assets / total assets,
- UTSMAO (share of fixed assets in total assets) = fixed assets / total assets,
- WPB (current ratio) = current assets / current liabilities,
- WZD (long term debt ratio) = long term liabilities / equity capital,
- WZKP (employment of working capital funds) = sales income / working capital,
- WZKW (debt equity ratio) = total liabilities / equity,
- WZO (total debt ratio) = total liabilities / total assets,
- WZZO (total involvement liabilities ratio) = sales income / total liabilities,
- UZAO (share of inventories in total assets) = total inventories / total assets,
- ZSZ(ability to repay a debt) = (net income + depreciation) / total liabilities.

Further reduction of independent variables was carried out on the stage of model estimation. For LFD algorithm there were stepwise backward and forward method used. For the construction of changeable qualities methods there was those ratios chosen, which have the highest discriminatory power.

For the construction of linear discriminant function there were following values included:

- Tolerance T factor equals 0,01 indicating which percentage of information about the enterprise (in this case at least 1%) must emend the feature to be able to enter the discriminant function equation,
- Critical value of F statistic as the basis for the introduction of variables at level 1.

By the result of the backward stepwise analysis was obtained an LFD\_1 model. Finally in the equal-pared for the linear discriminant function there were 5 variables included: WZZO, UTSMAO, ZSZ, WZKP and RS. The following correlation has been obtained:

$$\text{LFD}_1 = 0,327 \cdot \text{WZZO} + 3,276 \cdot \text{UTSMAO} + 0,402 \cdot \text{ZSZ} - 0,001 \cdot \text{WZKP} + 0,002 \cdot \text{RS} - 1,989.$$

Among the ratios, which have occurred to be important in the construction of the above model, four of them (WZZO, UTSMAO, ZSZ and RS) have a positive impact on the value of LFD\_1 function. This means that increase of total liabilities concerning with increment of sales revenue, the level of assets, the ability to repay debt, which depends on the net profit and return on sales affects the proper financial standing of each individual. Such dependence is consistent with the economic *trade off*.

WZKP ratio impact negatively on the LFD\_1 value, which is not covered in the theory of economics. To assess the impact of each variable on the LDF discrimination power there can be a model with standardized ratios used. It is presented in the below equation:

$$\text{LFD}(\text{ST})_1 = 0,821 \cdot \text{WZZO} + 0,769 \cdot \text{UTSMAO} + 0,349 \cdot \text{ZSZ} - 0,284 \cdot \text{WZKP} + 0,23 \cdot \text{RS}.$$

The biggest influence for the LFD\_1 function creation as well as its ability to distinguish classes has variable WZZO and UTSMAO. An assessment of the classification by the LFD\_1 is shown in the matrix in Table 2.

Table 2. Classification matrix for LFD\_1  
Tabela 2. Macierz klasyfikacji dla LFD\_1

Specification	Class 0 from model	Class 1 from model	% of accurate classifications
Actual 0 class	35	6	85,37
Actual 1 class	3	38	92,68
Total	38	44	89,02

Source: Own work.  
Źródło: Badanie własne.

The total percentage of proper classified companies is 89,02%, in which the better classified are companies with the well financial condition (38 of proper classifications for 41 objects from the sample). LFD\_1 have recognized 6 bankruptcies companies improperly grouping them in the units of correct standing, and 3 'healthy' objects classifying them as bankrupts.

There were also subjected to test a construction of LFD\_2 model using algorithm of stepwise backwards analysis. There were build a model with all the variables (financial ratios) and then in successive stages removed variables that affect a small extend on the discriminant function ability. Finally in the equitation there were nine variables included: WPB, WZO, WZD, ROA, WZKP, WZZO, UTSMO, UZAO and ZSZ).

The equitation of LFD\_2 model for original ratios looks like below:

$$\begin{aligned} \text{LFD}_2 = & 0,00196 \cdot \text{WPB} - 0,06 \cdot \text{WZO} + 0,09 \cdot \text{WZD} - 0,333 \cdot \text{ROA} - 0,0013 \cdot \\ & \cdot \text{WZKP} + 0,37 \cdot \text{WZZO} + 3,596 \cdot \text{UTSMO} + 1,726 \cdot \text{UZAO} + 0,842 \cdot \\ & \cdot \text{ZSZ} - 2,394. \end{aligned}$$

For standardized ratios the formula looks like below:

$$\begin{aligned} \text{LFD(ST)}_2 = & 0,345 \cdot \text{WPB} - 0,72 \cdot \text{WZO} + 0,48 \cdot \text{WZD} - 0,842 \cdot \text{ROA} - 0,311 \cdot \\ & \cdot \text{WZKP} + 0,815 \cdot \text{WZZO} + 0,845 \cdot \text{UTSMO} + 0,395 \cdot \text{UZAO} + \\ & + 0,732 \cdot \text{ZSZ} \end{aligned}$$

which shows that the biggest impact on the model on value of the discriminant function have the UTSMO ratio. Classification based on LFD\_2 received from the stepwise backward method gave the worse results than LFD\_1 model. The evaluation of the correctness of classification can be traced in the accuracy array placed in Table 3.

Table 3. Classification matrix fro LFD\_2  
Tabela 3. Macierz klasyfikacji dla LFD\_2

Specification	Class 0 from model	Class 1 from model	% of accurate classifications
Actual 0 class	35	6	85,37
Actual 1 class	4	37	90,24
Total	39	43	87,80

Source: Own work.  
Źródło: Badanie własne.

The total percentage of proper classified enterprises is 87,80%, but the recognition of companies with the good financial standing is better with the LFD\_2 method as well (4 wrong classifications). In the bankrupts there were 6 wrong classifications.

From the binary variables models group there were made an estimation of binomial logit model. For estimating the notes equation parameters there were used a features obtained from the discriminant analysis, characterized by the high ability to distinguish between classes.

For the LFD\_1 model those are the ratios: WZZO, UTSM AO, ZSZ, RS and for LFD\_2: WPB, WZO, WZD, ROA, WZKP, WZZO, UTSM AO, UZAO, ZSZ.

On the basis of the variables included in LFD\_1 there were a try to build a logit model, however the estimation process in this case did not complete properly – it was not possible to reach the convergence criterion for likelihood function. (plausibility function). It was decided than to remove RS variables and WZKP, because they discriminate the classes of the chosen enterprises on the lowest level (based on LFD\_1 estimation). The logit model LOG\_1 estimation results for 3 variables: WZZO, UTSM AO and ZSZ are shown in the Table 4.

Table 4. Logit model LOG\_1 estimation results for 3 ratios from LFD\_1  
Tabela 4. Wyniki estymacji modelu logitowego LOG\_1 dla 3 wskaźników z LFD\_1

Variable	Ratio	Standard error	T-Student statistic	P-Value
const	-2,37876	1,11392	-2,112	0,0347
WZZO	0,29496	0,22269	1,288	0,1979
UTSM AO	4,15609	1,78912	2,318	0,0205
ZSZ	19,22250	6,48012	3,038	0,0024

Source: Own work.

Źródło: Badanie własne.

Statistically significant ratios at 5% are ZSZ and UTSM AO. No statistical significance of parameters is a phenomenon often appears in models, in which the role of interpreter variable is played by binary variable (see Gruszczński [2003]). A positive estimate of LOG\_1 model structural parameters shows that increasing of the value of above features has a beneficial effect on the financial situation of the company. This model classified correctly 76 enterprises, which gives 92,7%: respectively 37 from the bankrupts and 39 with the proper financial condition. The classification matrix for LOG\_1 model is shown on Table 5.

Table 5. Classification matrix for the logit model LOG\_1  
Tabela 5. Macierz klasyfikacji dla modelu logitowego LOG\_1

Specification	Class 0 from model	Class 1 from model	% of accurate classifications
Actual 0 class	37	4	90,24
Actual 1 class	2	39	95,12
Total	39	43	92,70

Source: Own work.

Źródło: Badanie własne.

Since the discriminant function LFD\_2 was characterized by a worse accuracy and the total percentage of correct classifications than LFD\_1, it was decided to resign from building a logit model for the ratios used in its construction.



Accuracy equal 92,70% in the test sample numbering 82 objects for LOG\_1 is not satisfactory in the author's opinion, therefore the began to seek for the better set of interpretative variables has started, which would serve for the building of model for dichotomous variables. In this regard there was a modeling strategy *from general to specific* used, testing which set of variables will give the best classification results.

Logit model LOG\_2 estimation results for this group of ratios are shown in table 6.

Table 6. Logit model LOG\_2 estimation results for 11 ratios  
Tabela 6. Wyniki estymacji modelu logitowego LOG\_2 dla 11 wskaźników

Variable	Ratio	Standard error	T-Student statistic	P-Value
const	-17,69160	13,9613	-1,267	0,2051
UTSMAO	13,63250	13,6434	0,999	0,3177
WZZO	1,14195	0,7528	1,517	0,1293
URSKAO	10,21780	6,5730	1,555	0,1201
UNAO	2,93368	9,7527	0,300	0,7636
WPB	-0,00509	0,0287	-0,177	0,8591
UNSMO	36,65000	22,2734	1,645	0,0999
ZSZ	56,65830	24,3495	2,327	0,0200
UZAO	19,83010	16,4282	1,207	0,2274
OPZD	7,57613	4,9297	1,537	0,1243
WZKP	-0,02230	0,0212	-1,047	0,2949
WZKW	0,37348	0,2249	1,660	0,0968

Source: Own work.

Źródło: Badanie własne.

The negative notes of parameters staying on the variables WPB and WZKP indicate that lowering the current liquidity and the level of net current assets engagement should have a negative impact on the company's financial condition. The increase of other ratios included in LOG\_2 model will influence positively on the financial standing of enterprise.

In the model above only ZSZ ratio is the ratio significantly different from zero at 5% level. For the above model, the classification matrix is presented in Table 7.

Table 7. Classification matrix for the logit model LOG\_2  
Tabela 7. Macierz klasyfikacji dla modelu logitowego LOG\_2

Specification	Class 0 from model	Class 1 from model	% of accurate classification
Actual class 0	40	1	97,56
Actual class 1	2	39	95,12
Total	42	40	96,34

Source: Own work.

Źródło: Badanie własne.

This model is characterized by a relatively high overall percentage of correct classification (96,34%). Only in the case of one enterprise, which was declared as a bankrupt, there were an incorrect assignment, assigning it a value of 1 and for 2 companies from the

sample of enterprises with proper financial standing there were financial problems that could lead into a bankruptcy recognized (for this models there was a 0 class assigned, and actually they belonged to a 1 class).

It was also done a verification of models in validation sample, which was built on the basis of the set of expertly selected 16 enterprises. The group of bankrupt companies include: Techmex S.A., Monnari Trade S.A., Zakłady Naprawcze Taboru Kolejowego Łapy, Krośnieńskie Huty Szkła (for this objects in 2009 the commercial courts had declared bankruptcy), Odlewnie Polskie, Pronox Technology S.A. (bankruptcy with a possibility to an arrangement), Centrozap S.A. (judgment of 2004) and Próchnik S.A. The group of the 'healthy' enterprises includes: Optimus S.A., Lubawa S.A., Optopol Technology S.A., Stalprodukt S.A., Comp S.A., Doradztwo Gospodarcze DGA S.A., Relpol S.A. and Wojas S.A.

Validation was carried out for four models: LFD\_1, LFD\_2, LOG\_1 and LOG\_2, and the results of the classification are presented in the table 8. Models estimated on the basis of discriminant analysis in validation sample are characterized by the classification which does not offer a significantly bigger accuracy than the random one (for LFD\_1 the total percentage of proper classification is equal to 50% – together 8 properly recognized enterprises, for LFD\_2 this ratio stands at 55,56% – 10 properly recognized enterprises). Both linear discriminant functions in a similar manner identify individual companies included to a validation sample, but do not recognize properly defined classes.

Table 8. Classification matrix for models in validation sample  
Tabela 8. Macierz klasyfikacji dla model w próbie walidacyjnej

Group	Name of enterprise	LFD_1	LFD_2	LOG_1	LOG_2
Bankrupts	Techmex	0	0	1	1
	Monnari Trade	1	1	0	0
	ZNTK	1	1	0	0
	Krośnieńskie Huty Szkła	0	0	0	0
	Odlewnie Polskie	1	1	0	0
	Pronox Technology	0	0	0	0
	Centrozap	0	0	0	0
	Próchnik	1	1	0	1
'Healthy' enterprises	Optimus	1	0	1	1
	Lubawa	1	1	1	1
	Optopol Technology	0	1	1	1
	Stalprodukt	1	1	1	1
	Comp	1	1	1	1
	DGA	0	0	1	1
	Relpol	1	1	1	1
	Wojas	0	1	0	1

Source: Own work.

Źródło: Opracowanie własne.

In case of logit models verification in the validation sample the obtained results was much better. LOG\_1 model recognized improperly two enterprises: Techmex S.A. included into a bankrupts group and Wojas S.A. from the group of enterprises with good financial standing. The total percentage of the accurate classifications of LOG\_1 model is 88,89%. LOG\_2 model has recognized properly 15 companies, the only wrong classification was made with the Techmex S.A. company – the percentage of the proper classification for LOG\_2 is 94,4%. Results of logit models verification met the expectations. Basing on the relevancy of matrix from table 8 it can be said that logit models fared considerably better with the problem of enterprises classification than a linear discriminant function.

## SUMMARY

The result of the research was to build the classification models to predict corporate bankruptcy. An accurate forecast could serve as an information function both for managers, executive board and for banks or other financial sector units.

This thesis presents several approaches to the concept of building the classification models. The starting point was an appropriate choice of bankruptcy predictors, which would divide two studied groups of objects: companies 'healthy' and bankrupts. In this study the selection of interpreter variables was based on the features-financial ratios matrix analysis. For the analysis there were 15 exogenous variables used, and their final selection for models was based on the own values of standardized discriminant function estimations.

Based on the research there are better results for use of the classification issue with logit models over the discriminant linear function. This means that the task of companies classification into groups: bankrupts, non-bankrupts is a nonlinear issue. LOG\_1 model has a high accuracy of the forecasts (92,7% of correct classifications in the base sample). LOG\_2 was estimated on the basis of interpreter variables, which were chosen in accordance with the modeling strategy *from general to specific*. For this model counted R-square was 96,34%. It should also be clear that logit models in a very reliable way solves the tasks of bankruptcy prediction in the validation samples – model LOG\_1 classified correctly 88,89% of companies, while LOG\_2 classified correctly 94,4%.

## REFERENCES

- Altman E.I., 1968. Financial Ratios, Discriminant analysis and the prediction of corporate bankruptcy. *Journal of Finance*, Vol. 23, No. 4.
- Gajdka J., Stos D., 2003. Ocena kondycji finansowej polskich spółek publicznych w okresie 1998–2001. [in:] Zarzecki D. (ed.) *Czas na pieniądz, Zarządzanie finansami, Mierzenie wyników i wycena przedsiębiorstw*, t. 1. Wydawnictwo Uniwersytetu Szczecińskiego. Szczecin.
- Gasza R., 1997. Związek między wynikami analizy typu Altmana a kształtowaniem się kursów akcji wybranych spółek giełdowych w Polsce. *Rezultaty badań najstarszych spółek giełdowych w latach 1991–1995*. *Bank i Kredyt*, Nr 3.

- Gruszczyński M., 2003. Modele mikroekonometrii w analizie i prognozowaniu zagrożenia finansowego przedsiębiorstw. Instytut Nauk Ekonomicznych Polskiej Akademii Nauk, Nr 34, Warszawa.
- Hadasik D., 1998. Upadłość przedsiębiorstw w Polsce i metody jej prognozowania. Akademia Ekonomiczna w Poznaniu, Zeszyty Naukowe – seria II.
- Hołda A., 2001. Prognozowanie bankructwa jednostki w warunkach gospodarki polskiej z wykorzystaniem funkcji dyskryminacyjnej  $Z_H$ . Rachunkowość nr 5/2001.
- Kisielińska J., 2008. Modele klasyfikacyjne prognozowania sytuacji finansowej gospodarstw rolniczych. Wydawnictwo SGGW, Warszawa.
- Kisielińska J., Waszkowski A., 2010. Polskie modele do prognozowania bankructwa przedsiębiorstw i ich weryfikacja. Ekonomika i Organizacja Gospodarki Żywnościowej. Zeszyty Naukowe nr 82, SGGW w Warszawie.
- Koralun-Bereźnicka J., 2006. Ocena możliwości wykorzystania wybranych funkcji dyskryminacyjnych w analizie polskich spółek giełdowych. Studia i Prace Kolegium Zarządzania i Finansów. Zeszyt Naukowy nr 69, SGH w Warszawie.
- Maddala G.S., 2004. Limited – dependent and qualitative variables in econometrics. Cambridge University Press, Cambridge.
- Mączyńska E., Zawadzki M.: Dyskryminacyjne modele predykcji bankructwa przedsiębiorstw. Ekonomista 2/2006.
- Mikroekonometria, red. Gruszczyński M., 2010. Wolters Kluwer, Warszawa.
- Rogowski W., 1999. Możliwość wczesnego rozpoznawania symptomów zagrożenia zdolności płatniczej przedsiębiorstwa. Bank i Kredyt, Nr 6.
- Stasiewski T., 1996. Z-score – indeks przewidywanego upadku przedsiębiorstwa. Rachunkowość, Nr 12.

## MODELE KLASYFIKACJI WZORCOWEJ W PREDYKCJI UPADŁOŚCI PRZEDSIĘBIORSTW

**Streszczenie.** W pracy dokonano próby stworzenia modeli klasyfikacji wzorcowej, które umożliwiłyby przewidywanie upadłości przedsiębiorstw. Przedstawiono ekonomiczne aspekty bankructwa jednostek gospodarczych oraz jego przyczyny. Uwagę poświęcono metodom predykcji upadłości ze szczególnym uwzględnieniem analizy dyskryminacyjnej oraz modeli logitowych. Dokonano przeglądu literatury w zakresie istniejących modeli wczesnego ostrzegania. Punkt poświęcony badaniom własnym zawiera opis oraz wyniki estymacji oryginalnych modeli predykcji bankructwa. Zweryfikowano również otrzymane systemy wczesnego ostrzegania w próbie walidacyjnej.

**Słowa kluczowe:** prognozowanie upadłości, modele klasyfikacji, analiza dyskryminacyjna, model logitowy

Accepted for print – Zaakceptowano do druku 24.05.2011